ATMOSPHERIC AND OCEANIC PHYSICS, English Translation, VOL. 30, NO. 6, JUNE 1995
Russian Edition: NOVEMBER–DECEMBER 1994

Influence of parameters of coarsely-dispersed aerosols' microstructure on properties of light fields in medium and on optical transfer functions: Analytic solutions

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Abstract. A new analytic expression for optical transfer function (OTF) and spatial mutual coherence function (MCF) for light fields in randomly-inhomogeneous media containing coarse particles was obtained. The expression relates OTF and MCF directly with the size distribution function of the particles and optical constants of their substance. Solution of the problem is based on the consideration of the optics of individual particles together with the theory of radiative transfer in small-angle approximations. Comparison with the calculations by the Monte Carlo method was carried out.

1. Introduction

The characteristics of light fields in clouds and fogs, dust escapes and other coarsely-dispersed aerosols are of significant interest in solving the problems of atmospheric optics, climatology, long sight, sounding and optical communication under conditions of high atmospheric turbidity. At relatively small optical thicknesses of the layers, on adequate method for their calculation is the small-angle approximation (SAA) in the theory of radiative transfer [*Dolin,* 1964; *Zege et al.,* 1991; *Ishimaru,* 1981].

As is known, in SAA the spatial-angular distribution of light field intensity $I(z, \mathbf{r}, \mathbf{n}_\perp)$ at a depth z from a spatially limited source characterized by the small angular divergence and intensity $I_0(z=0, \mathbf{r}, \mathbf{n}_\perp)$ (incident light is normal to the layer boundary) can be determined as the Fourier transformation of the function [*Dolin,* 1964; *Zege et al.,* 1991; *Ishimaru,* 1981]:

$$I(z, \mathbf{v}, \mathbf{p}) = I_0(\mathbf{v}, \mathbf{p} + \mathbf{v}z) J(z, \omega, p) \tag{1}$$

where

$$J(z, \omega, p) = \exp(-\tau(1 - \varphi(p, \omega))) \tag{2}$$

$$\begin{gathered} \varphi(p, \omega) = \frac{1}{\varepsilon} \int_0^1 \sigma(p + \omega s)\, ds \\ \sigma(p) = \frac{1}{2} \int_0^\infty \sigma(\theta) J_0(p\theta)\theta\, d\theta \end{gathered} \tag{3}$$

Here $\tau = \varepsilon z$ is the optical thickness; $\sigma(\theta)$ and ε are the coefficients of directional light-scattering (CDL) and extinction; $\mathbf{p}$ and $\mathbf{v}$ are the angular and spatial frequencies, respectively; $I_0(\mathbf{v}, \mathbf{p} + \mathbf{v}z)$ is the Fourier transform (FT) of the spatial-angular distribution of the source brightness; $J_0(p\theta)$ is the Bessel function of the zeroth order; $J(z, \omega, p)$ is the FT of the Green's function for the problem; and $\omega = |\mathbf{v}|z$; $p = |\mathbf{p}|$. In (1)–(3) the axial symmetry of the media about the axis of the beam is assumed.

The aim of this work is to find simple analytical formulas relating FT $J(z, p, \omega)$ with the sizes and refractive index $m = n - i\chi$ of coarse ($x \gg 1$, $x = ka$, $k = 2\pi/\lambda$, a is the radius of particles, λ is the wavelength) discrete scatterers. The problem of calculation of the spatial-angular distribution of intensity $I(z, \mathbf{r}, \mathbf{n}_\perp)$ with these formulas reduces to the Fourier transform of a simple analytic function. In passing, the analytic expressions for the optical transfer function (OTF) of a medium and the spatial mutual coherence function (MCF) of a light field can be obtained. According to *Zege et al.* [1991], the OTF of a medium is:

$$S(\omega) = J(z, \omega, p = 0) \tag{4}$$

and according to *Zege et al.* [1991], *Ishimaru* [1981], *Ovchinnikov and Tatarskiy* [1972], and *Lutomirski* [1978], the MCF of a light field in a medium $\Gamma(R)$ is:

$$\Gamma(r) = J(z, \omega = 0, p = R) \tag{5}$$

where $R = kr$, r is the distance between two points in a medium.

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2. Relation of Fourier Transform of the Green's Function With the Microphysical Parameters of a Layer

The analytic relation of FT of the Green's function $J(z,\omega,p)$ (2) with the sizes of particles and their complex refractive index is determined below. At first, a monodispersed medium is considered. In the Appendix the small-angle approximation is obtained for the coefficient of directional light-scattering (CDL) $\sigma(\theta)$ in a medium containing the coarsely-dispersed scatterers. From formula (A9) for the Fourier transform of CDL $\sigma(p)$ (3) it follows:

$$\sigma(p) = \sum_{j=1}^{3} \sigma_j(p) \tag{6}$$

where

$$\sigma_1(p) = 2\Sigma U(D)(\arccos D - D\sqrt{1-D^2})/\pi \tag{7}$$

$$\sigma_2(p) = \frac{\alpha\Sigma}{2(\alpha^2+p^2)^{3/2}} \tag{8}$$

$$\sigma_3(p) = \frac{\gamma\Sigma}{4\beta^*} e^{-p^2/4\beta^* - c} \tag{9}$$

Here $D = p/2x$, $\Sigma = N\pi a^2$, $U(v) = 1$ at $v < 1$ and in other cases $U(v) = 0$. Parameters α, β^*, γ, and c depend on sizes of the particles and their complex refractive index. These parameters are given in the Appendix.

Substitution of (6) into (3) (taking into account that for coarse particles $\varepsilon = 2\Sigma$ [*Deirmendjian,* 1969; *Shifrin,* 1951]) yields:

$$\varphi(p,\omega) = \sum_{j=1}^{3} \varphi_j(p,\omega) \tag{10}$$

$$\begin{aligned}\varphi_1(p,\omega) &= \frac{2x}{\pi\omega} U(\frac{p}{2x}) \times \\ &\times \left[W(\frac{p+\omega}{2x}) U(\frac{p+\omega}{2x}) - W(\frac{p}{2x}) \right]\end{aligned} \tag{11}$$

$$\varphi_2(p,\omega) = \frac{1}{4\omega\alpha} \left(\frac{p+\omega}{\sqrt{\alpha^2+(p+\omega)^2}} - \frac{p}{\sqrt{\alpha^2+p^2}} \right) \tag{12}$$

$$\begin{aligned}\varphi_3(p,\omega) &= \frac{\gamma e^{-c}}{8\omega} \sqrt{\pi/\beta^*} \times \\ &\times \left[\mathrm{erf}\left(\frac{p+\omega}{2\sqrt{\beta^*}}\right) - \mathrm{erf}\left(\frac{p}{2\sqrt{\beta^*}}\right) \right]\end{aligned} \tag{13}$$

where

$$\begin{aligned} W(\xi) &= \xi \arccos \xi - (2+\xi^2)\sqrt{1-\xi^2/3} \\ \mathrm{erf}(\xi) &= 2/\sqrt{\pi} \int_0^{\xi} e^{-s^2}\, ds \end{aligned} \tag{14}$$

Formulas (2), (10)–(13) give the solution of the problem, explicitly relating the FT of the Green's function $J(z,\omega,p)$ with a diffraction parameter x, refractive n and absorption χ indexes. The latter is involved in the solution in the combination $c = 4\chi x$ only, which is caused by the condition $\chi \ll n$ frequently realized in the optical spectral region. Components $\sigma_i(p)$ of FT of CDL $\sigma(p)$ correspond to the individual contributions of diffraction ($i = 1$), reflection ($i = 2$) and refraction ($i = 3$) of light by a particle. The influence of these three physical processes on OTF in the case of light scattering by a coarse particle is taken into account by the functions $\varphi_j(\omega)$.

Function $L = -\log J(z,\omega,p) = \tau(1-\varphi(p,\omega))$ depends linearly on characteristics of unit volume, i.e., it is additive relative to contributions of separate scatterers and each fraction as a whole [*Zege et al.,* 1991]. Hence it follows:

1. To extend these results to the case of polydispersed media, it will suffice to integrate the function L together with the particle size distribution function (DF) $f(a)$; so it is deduced

$$J(z,\omega,p) = \exp\left(-\int_0^{\infty} Lf(a)\, da \right) \tag{15}$$

instead of (2).

2. For a mixture of M different substances it can be written

$$J(z,\omega,p) = \exp\left(-\sum_{i=1}^{M} L_i \right) \tag{16}$$

where the value L_i relates to the ith component of a mixture.

The detailed analysis of relations (2), (10)–(13) is given below. For this purpose three transitions to the limits $p \to 0$, $\omega \to 0$ and $p \to 0$ with $\omega \to 0$ are considered sequentially. These lead, respectively, to the analytic solutions for the optical transfer function of a dispersed layer, for the diffuse transmittance coefficient, and for the mutual coherence function of an electromagnetic field in a dispersed media as well as for FT of the Green's function in the problem of the wide beam propagation.

3. Optical Transfer Function of a Layer

At $p = 0$ from (4), (2), (10) it follows

$$S(\omega) = e^{-\tau(1-\varphi(\omega))} \tag{17}$$

where $\varphi(\omega) = \varphi(0, \omega)$ and

$$\varphi(0,\omega)=\frac{1}{4\alpha\sqrt{\alpha^2+\omega^2}}+\frac{\gamma e^{-c}}{8\omega}\sqrt{\frac{\pi}{\beta^*}}\operatorname{erf}\left(\frac{\omega}{2\sqrt{\beta^*}}\right)+ +\frac{1}{\pi l}\{l\arccos l-\frac{(2+l^2)\sqrt{1-l^2}}{3}\}U(l)+\frac{2}{3\pi l} \tag{18}$$

Here $l = \omega/2x$. At $1 \ll \omega < 2x$ it follows from (18):

$$\varphi(\omega) = \frac{1}{4\alpha\omega} + \frac{\gamma e^{-c}}{8\omega}\sqrt{\frac{\pi}{\beta^*}} + \frac{4x}{3\pi\omega} \tag{19}$$

Let us estimate contributions of the different summands to (19). Relations of the first and second summand to the third one, the last being determined by diffraction on the particle's outer surface, are equal to $\xi_1 = 3\pi/16\alpha x$ and $\xi_2 = (3\pi\gamma e^{-c}/32x)\sqrt{\pi/\beta^*}$, respectively. For example, for water droplets ($m = 1.33$), these relations lead to $\xi_1 = 0.2/x$ and $\xi_2 = 3.75/x$. So, for the values of parameter $x \geq 60$, specific for clouds and fogs [*Deirmendjian,* 1969; *Shifrin,* 1951] (in the visible spectral range), $\xi_1 \leq 0.003$, $\xi_2 \leq 0.006$. Hence it follows that the asymptotic behavior of the function $\varphi(\omega)$ at high frequencies is primarily determined by the diffraction component (see also *De Wolf* [1978], *Borovoy et al.* [1986], and *Zege and Kokhanovskiy* [1992]). Thus at $\omega \gg 1$ it may be written:

$$\varphi(\omega) = \frac{4x}{3\pi\omega} \tag{20}$$

instead of (19).

Relation (20) can be used as a basis for the calculation of radii of coarse particles in multiply scattering media [*Zege and Kokhanovskiy,* 1992]. At $\varphi < 10^{-2} (\omega \geq 40x)$ the dependence on ω in (17) can be neglected and then

$$S = e^{-\tau} \tag{21}$$

that is, OTF is determined by the unscattered light only [*Zege et al.,* 1991]. In the range of small spatial frequencies ($\omega \to 0$) it follows from (18), the terms $\approx x^{-1}$ being neglected:

$$\varphi(\omega) = \varphi(0) - A\omega^2 \tag{22}$$

where

$$\varphi(0) = \frac{1}{2} + \frac{1}{4\alpha^2} + \frac{\gamma e^{-c}}{8\beta^*}, \quad \Lambda = \frac{\gamma e^{-c}}{96\beta^*} + \frac{1}{8\alpha^4} \tag{23}$$

Function

$$S(0) = e^{-\tau(1-\varphi(0))} \tag{24}$$

is a coefficient of diffuse transmittance at normal illumination of the layer by a wide beam $t(\tau, \mu_0 = 1)$, where $\mu_0 = \arccos\vartheta_0$, and ϑ_0 is the angle of incidence of a beam on a layer. Let us consider it in greater detail. As follows from (24), (23):

$$t(\tau,\mu_0{=}1) = \exp\{-\tau\left(\frac{1}{2} - \frac{1}{4\alpha^2} - \gamma\frac{e^{-c}}{8\beta^*}\right)\} \tag{25}$$

where for coarse particles $\tau = 2\Sigma z$ [*Shifrin,* 1951].

Function (25) gives the analytic relation of the coefficient of diffuse transmittance of light with the parameters of microstructure of a layer. In SAA of the transfer theory, the following relation is known [*Zege et al.,* 1991]:

$$t(\tau,\mu_0{=}1) = \exp\{-\tau(1-\Lambda^*)\} \tag{26}$$

where $\Lambda^* = \Lambda F(\theta_0)$ is the effective albedo of unit volume of a medium; Λ is the true albedo; and $F(\theta_0) = \int_0^{\theta_0} x(\theta)\sin\theta\, d\theta$, $x(\theta)$ is the scattering phase function. Empirical parameter Λ^* in SAA is not determined exactly; it depends on the chosen angle restriction on the small-angle peak θ_0 [*Zege et al.,* 1991] (it is usually assumed that $\theta_0 = \pi/4$ and in quasi-single scattering $\theta_0 = \pi/2$). Within the frames of the given approach, a simple relation of this parameter with the characteristics of particles can be obtained (see (25)):

$$\Lambda^* = \frac{1}{2}\{1 + \frac{1}{2\alpha^2} + \frac{\gamma e^{-c}}{4\beta^*}\} \tag{27}$$

4. Mutual Coherence Function

The case $\omega = 0$ is considered below. The function (see *Zege et al.* [1991])

$$J(p) = \exp(-\tau + z\sigma(p)) \tag{28}$$

is the Fourier transform of the normalized intensity $I(\tau, \theta)$ provided the medium is illuminated by an infinitely wide beam. The intensity

$$I(\tau,\theta) = 2\pi\int_0^\infty J(p)J_0(p\theta)p\,dp \tag{29}$$

and function $\sigma(p)$ is determined by formulas (6)–(9).

Relation (28) is also of interest irrespective of the calculation of integral (29), since after substitution [*Zege et al.,* 1991; *Ishimaru,* 1981; *Ovchinnikov and Tatarskiy,* 1972; *Lutomirski,* 1978] of $R = kr$ for p (see (5)) it gives the analytic solution for a mutual coherence function

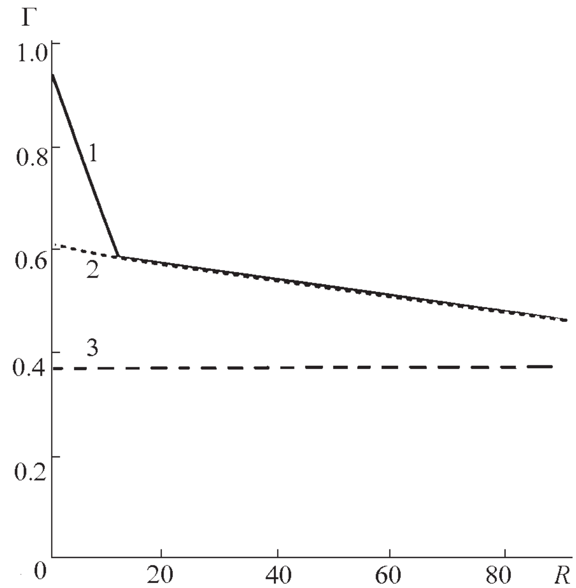

Figure 1. Dependence $\Gamma(R)$ at $\tau = 1$, $x = 100$, $n = 1.33$, $\chi = 0$ (1, refracted field is taken into account (see (4.3)); 2, refracted field is not taken into account [*Borovoy et al.,* 1986]; and 3, asymptotics of $\Gamma(R)$ at $R \gg 1$).

$\Gamma(r)$ of an electromagnetic field in a dispersed medium (an incident wave is a plane one):

$$\Gamma(r) = \exp(-\tau(1 - \psi(r))) \tag{30}$$

where

$$\psi(r) = \sigma(R)/2\Sigma \tag{31}$$

The coefficient of spatial correlation $K(r) = \Gamma(r)/\Gamma(0)$ and the spatial function of correlation of intensity in Fraungoffer's zone behind a layer $M(r) = \Gamma^2(r) - e^{-2\tau}$ [*Borovoy et al.,* 1986] is expressed by the function $\Gamma(r)$.

MCF of a scattering layer was considered earlier by *Lutomirski* [1978], *De Wolf* [1978], and *Borovoy et al.* [1986]. Usually coherent Γ_k and incoherent $\Gamma_n(r)$ components of MCF are separated [*Ishimaru,* 1981]:

$$\Gamma(r) = \Gamma_k + \Gamma_n(r) \tag{32}$$

where $\Gamma_k = e^{-\tau}$, $\Gamma_n(r) = e^{-\tau}(e^{\tau\psi(r)} - 1)$.

At $r \to \infty$, as was to be expected, $\psi(r) \to 0$ and $\Gamma(r) \to \Gamma_k(r)$.

Function (30) was considered by *Borovoy et al.* [1986], a diffraction term in $\sigma(R)$ (the first term in (6)) being exclusively taken into account. This work considers, additionally, the terms concerning the geometry-optical part of a scattered field. Their influence is of particular importance for the ranges of small r values: it is these terms that determine the behavior of MCF $\Gamma(r)$ at $r \to 0$. Indeed, let us expand the function $J_0(R\theta)$ in a series relative to the small parameter $R\theta$ and restrict ourselves to the first two terms. Then

$$\begin{aligned} \psi(r) &= \psi(0) - \theta_2 \Lambda^* R^2/4 \\ \Lambda^* \theta_2 &= \frac{3}{2\alpha^4} + \frac{\gamma e^{-c}}{8\beta^{*2}} \end{aligned} \tag{33}$$

The parameter θ_2 is the average value of the squared scattering angle. This parameter is primarily determined by the geometry-optical component of the scattered field. The function $\psi(0) = \varphi(0)$ (see (23)). Therefore $\Gamma(0) = S(0) = t(\tau, \mu_0 = 1)$, as it must be in accordance with *Zege et al.* [1991].

From (30), (33) it follows that at small R values MCF has the Gaussian form, namely

$$\Gamma(r) = \Gamma(0) e^{-(r/s)^2} \tag{34}$$

where s is the distance at which MCF is attenuated by the factor e:

$$s = A\lambda/\sqrt{\tau}, \quad A = 1/\pi\sqrt{\theta_2 \Lambda^*} \tag{35}$$

In effect, relation (34) is a consequence of the azimuth independence of the scattering phase function.

The distance s decreases with increasing optical thickness of a medium as $1/\sqrt{\tau}$. This result was pointed out earlier by *Lutomirski* [1978], the spherical incident wave being considered. The A value is independent of the size of particles and is determined by the refractive index n only, if the particles are rather coarse and do not absorb the light ($\chi = 0$). The A value varies from 1 at $n = 1.33$ to 0.5 at $n = 1.64$. For watersols ($m = 1.33$, $x \ll 1$), $s < \lambda$ at $\tau > 1$. A rise in absorption causes the parameter θ_2 to decrease and the s value to increase. Figure 1 presents MCF for water droplets ($x = 100$), the function being calculated by the formulas (30) and (6) (curve 1). In the same figure the asymptote of (30) is given for $R \gg 1$ (curve 3). Curve 2 is calculated only with inclusion of the diffraction component of scattering, as was done by *Borovoy et al.* [1986]. It is seen that at $r \gg \lambda$, MCF is completely determined by the diffraction component; the function slowly decreases with characteristic parameter $s \sim a/\sqrt{\tau}$ (see *Lutomirski* [1978]). The inclusion of the component $\Gamma(0)$ is very significant, as it is the component that describes the abrupt MCF change at $r < \lambda$. Actually, of the rigid particles that do not absorb radiation, about a half scatter light incoherently.

Figure 1 gives MCF at $\tau = 1$. In SAA, MCF at any depth τ can be obtained using the equality:

$$\Gamma(R, \tau) = [\Gamma(R, 1)]^\tau \tag{36}$$

(see (30)).

5. Polydispersed Media

The extension of these results to polydispersed media is simple (see (15)). For example, in the case of polydispersed media, assuming $\beta^* \approx \beta$, the principal expressions (17), (25), (30) are replaced by

$$S(\omega) = \exp(-\tilde{\tau}[1-\tilde{\varphi}(\omega)]) \quad (37)$$

$$t(\tau, \mu_0 = 1) = \exp(-\tilde{\tau}[1-\tilde{\varphi}(0)]) \quad (38)$$

$$\Gamma(R) = \exp(-\tilde{\tau}[1-\tilde{\psi}(R)]) \quad (39)$$

Here

$$\tilde{\tau} = 2N\pi M_2 z \quad (40)$$

$$\begin{aligned} \tilde{\varphi}(\omega) = \frac{1}{2}\{(1-Z_2) - \frac{2\delta a_{12}}{\pi}(1-Z_1) + \\ + \frac{\delta^3}{6\pi}a_{-12}(1-Z_{-1}) + \frac{4a_{32}Z_3}{3\pi\delta} + \\ + \frac{1}{2\alpha^2\sqrt{1+(\omega/\alpha)^2}} + \frac{\gamma d}{4\omega}\sqrt{\frac{\pi}{\beta}}\,\mathrm{erf}\left(\frac{\omega}{2\sqrt{\beta}}\right)\} \end{aligned} \quad (41)$$

$$\tilde{\varphi}(0) = \frac{1}{2} + \frac{1}{4\alpha^2} + \frac{\gamma d}{8\beta} \quad (42)$$

$$\begin{aligned} \tilde{\psi}(R) = \frac{1}{2}\{1 - \bar{Z}_2 - \frac{2\Delta a_{12}}{\pi}(1-Z_1) + \frac{2\Delta^3}{6\pi}a_{-12}\times \\ \times(1-\bar{Z}_{-1}) + \frac{\alpha}{2(\alpha^2+R^2)^{3/2}} + \frac{\gamma d}{4\beta}\exp(-\frac{R^2}{4\beta})\} \end{aligned} \quad (43)$$

where

$$\delta = \omega/2k, \quad \Delta = R/2k, \quad a_{ij} = M_i/M_j$$

$$M_i = \int_0^\infty a^i f(a)da, \quad d = M_2^*/M_2$$

$$M_j^* = \int_0^\infty a^j e^{-c} f(a)da,\ Z_j = \bar{M}_j/M_j,\ \bar{M}_j = \int_0^\delta a^j f(a)da$$

$$z_j = \tilde{M}_j/M_j, \quad \tilde{M}_j = \int_0^\Delta a^j f(a)da \quad (44)$$

For most practically used types of the size distribution functions, the integrals M_j, M_j^*, and $\bar{M}_j$ can be analytically calculated. For example, for Γ-distribution $f(a) = Ca^\mu e^{-\mu a/a_0}$ ($C = \mathrm{const}$) common to clouds and fogs, equations (40)–(43) are to be transformed:

$$\tilde{\tau} = 2N\pi a_0^2(1+1/\mu)(1+2/\mu)z \quad (45)$$

$$\begin{aligned} \tilde{\varphi}(\omega) = \frac{1}{2}\Big[1 + \frac{8\rho_{32}}{3\pi\omega}P(\mu+4,\Delta_1) - P(\mu+3,\Delta_1) - \\ - \frac{2\Delta_1}{\pi(\mu+2)}(1-P(\mu+2,\Delta_1)) + \\ + \frac{\Delta_1^3}{6\pi\mu(\mu+1)(\mu+2)}(1-P(\mu,\Delta_1)) + \\ + \frac{1}{2\alpha^2\sqrt{1+(\omega/\alpha)^2}} + \frac{\gamma d}{4\omega}\sqrt{\pi/\beta}\,\mathrm{erf}\,(\omega/2\sqrt{\beta})\Big] \end{aligned} \quad (46)$$

$$\tilde{\varphi}(0) = \frac{1}{2} + \frac{1}{4\alpha^2} + \frac{\gamma d}{8\beta} \quad (47)$$

$$\begin{aligned} \tilde{\psi}(R) = \frac{1}{2}\{1 - \frac{2\Delta_2}{\pi(\mu+2)}(1-P(\mu+2,\Delta_2)) - \\ - P(\mu+3,\Delta_2) + \frac{2\Delta_2^3(1-P(\mu,\Delta))}{3\pi(\mu+2)(\mu+1)\mu} + \\ + \frac{\alpha}{2(\alpha^2+R^2)^{3/2}} + \frac{\gamma d}{4\beta}\exp(-\frac{R^2}{4\beta})\} \end{aligned} \quad (48)$$

where $P(j,x) = \frac{1}{\Gamma(j)}\int_0^x e^{-t}t^{j-1}\,dt$ is the incomplete Γ-function, $\Gamma(j)$ is a gamma-function, and $d = (1+4\chi\rho_0/\mu)^{-(\mu+3)}$, $\Delta_1 = \mu\omega/2\rho_0$, $\Delta_2 = \mu R/2\rho_0$, $\rho_0 = ka_0$. If j is an integer [*Abramowitz and Stigun,* 1979], then

$$P(j,x) = 1 - e^{-x}\sum_{n=0}^{j-1}\frac{x^n}{n!} \quad (49)$$

These relations make it possible to investigate quickly and with high accuracy the dependencies of the properties of light fields on the microphysical parameters of coarsely-dispersed sols. As an illustration, Figure 2 shows OTF $S(\omega)$ relevant to the coarse-drop water aerosols, the curves being calculated by the formulas (37), (45), and (46) and by the Monte Carlo method [*Drofa and Usachev,* 1980]. The results obtained by the two methods practically coincide.

6. Appendix

This appendix derives a simple analytic formula for the coefficient of directional light-scattering (CDL).

As is known [*Deirmendjian,* 1969; *Shifrin,* 1951], CDL $\sigma(\theta)$ for the aerosol formed by polydispersed spherical particles is characterized by the dimensionless Mie's intensity $I(a,\theta)$:

$$\sigma(\theta) = N\int_0^\infty \sigma(a,\theta)f(a)\,da, \quad \sigma(a,\theta) = \frac{4\pi I(a,\theta)}{k^2} \quad \text{(A1)}$$

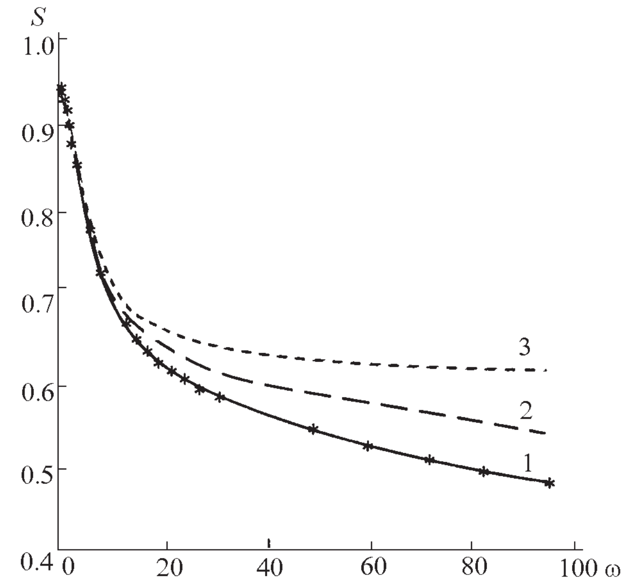

Figure 2. Dependencies $S(\omega)$ at different a_0 in the size distribution function ($a_0 = 4$ μm (1), 8 μm (2), 50 μm(3)); $\mu = 6$, $n = 1.33$, $\chi = 0$, $\lambda = 0.7$ μm (asterisks are the Monte Carlo method data, $a_0 = 4$ μm [*Drofa and Usachev,* 1980].

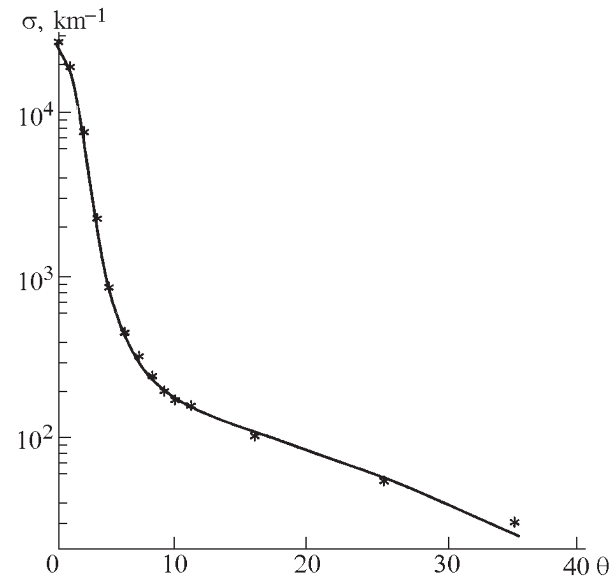

Figure 3. CDL $\sigma(\theta)$ for the model Cloud C1 [*Deirmendjian,* 1969] at $\lambda = 0.7$ μm (asterisks are exact calculations [*Deirmendjian,* 1969]; solid curve is calculated in approximation (A1)–(A9)).

where N is the number of particles in a unit volume of a medium, $k = 2\pi/\lambda$, $f(a)$ is the size distribution of particles, and $\int_0^\infty f(a)\,da = 1$.

Calculation of the function $I(a,\theta)$ for the coarse particles is difficult and requires great expenditures of the computer time. In this connection it is frequently calculated in the geometric optics approximation. In this approximation, as is known [*Shifrin,* 1951]:

$$I(a,\theta) = \sum_{j=1}^{\infty} I_j(a,\theta) \tag{A2}$$

where functions $I_j(a,\theta)$ are the beams' intensities (the subscripts $j = 1, 2, 3, \ldots$ correspond to the light diffracted by the particles, reflected from the surface of the particles, transmitted through the particles without any inner reflections, and so on).

In the angle range of interest ($\theta \ll 1$) the contributions of the functions $I_j(a,\theta)$ subscripted by $j \geq 4$ to the value $I(a,\theta)$ may be neglected [*Shifrin,* 1951]. In this case, as is known [*Shifrin,* 1951]:

$$I_1(a,\theta) = \frac{x^2 J_1^2(x\theta)}{\theta^2}, \quad I_2(a,\theta) = \frac{x^2 R(\theta)}{4}$$
$$I_3(a,\theta) = \frac{x^2 T(\theta)}{4} e^{-c\Delta(\theta)} \tag{A3}$$

Here $J_1(x\theta)$ is a Bessel function, the dimensionless diffraction parameter $x = ka$, $c = 4\chi\rho$,

$$R(\theta) = \frac{1}{2}\sum_{j=1}^{2}\left[\frac{N_j^2\sqrt{1-q^2} - \sqrt{n^2-q^2}}{N_j^2\sqrt{1-q^2} + \sqrt{n^2-q^2}}\right]^2 \tag{A4}$$

$$T(\theta) = \left(\frac{2n}{n^2-1}\right)\frac{(nq-1)^3(n-q)^3(1+q^4)}{2q^5(1+n^2-2nq)^2} \tag{A5}$$

$$\Delta(\theta) = \frac{n-q}{\sqrt{1+n^2-2nq}}, \quad q = \cos\frac{\theta}{2}$$
$$N_1 = 1, \quad N_2 = n^2 \tag{A6}$$

At $\theta \ll 1$ the following approximation is valid:

$$R(\theta) = e^{-\alpha\theta}, \quad T(\theta) = \gamma e^{-\beta\theta^2}$$
$$\Delta(\theta) = 1 - \frac{2n-1}{4(n-1)}\theta^2 \tag{A7}$$

where

$$\gamma = \frac{1}{(n-1)^2}\left(\frac{2n}{n+1}\right)^4 \tag{A8}$$

Table 1. Dependence of parameters α and β on the n value

n	1.33	1.4	1.53	1.6
α	3.0	2.7	2.4	2.3
β	4.7	3.6	2.4	2.0

This approximation simplifies the following calculations.

The constants α, β and γ depend exclusively on the real part of the complex refractive index n of the particles (see Table 1). The maximum error of approximation (A7) is less than 15% at $\theta \leq 35°$ and $n = 1.33$–1.6. It diminishes as n increases and as angle θ decreases.

The accuracy of the calculations of $\sigma(\theta)$ by the formula (A1) at

$$\sigma(a,\theta) = \{\frac{4J_1^2(\theta\rho)}{\theta^2} + e^{-\alpha\theta} + \gamma\exp(-\beta^*\theta^2 - c)\}\pi a^2$$
$$\beta^* = \beta + \frac{2n-1}{4(n-1)}c \tag{A9}$$

can be judged using Figure 3. This figure presents the scattering phase function $x(\theta)$ (multiplied by the value of scattering coefficient σ); the function was calculated by *Deirmendjian* [1969] for the model Cloud C1 at $\lambda = 0.7\,\mu$m and was obtained in the approximation (A1–A9).

As seen in Figure 3, the error of the approximation under consideration is no more than 15% at $\theta \leq 35°$; over a wide angle region it is even less than this value.

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(Received July 28, 1993.)